Petri-Net-Based Hypertext: Document Structure with Browsing Semantics

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We present a formal definition of the Trellis model of hypertext and describe an authoring and browsing prototype called αTrellis that is based on the model. The Trellis model not only represents the relationships that tie individual pieces of information together into a document (i.e., the adjacencies), but specifies the browsing semantics to be associated with the hypertext as well (i.e., the manner in which the information is to be visited and presented). The model is based on Petri nets, and is a generalization of existing directed graph-based forms of hypertext. The Petri net basis permits more powerful specification of what is to be displayed when a hypertext is browsed and permits application of previously developed Petri net analysis techniques to verify properties of the hypertext. A number of useful hypertext constructs, easily described in the Trellis model, are presented. These include the synchronization of simultaneous traversals of separate paths through a hypertext, the incorporation of access controls into a hypertext (i.e., specifying nodes that can be proven to be accessible only to certain classes of browsers), and construction of multiple specialized (tailored) versions from a single hypertext.

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1. INTRODUCTION

Hypertext is a structuring mechanism for information, one that is particularly well suited for use on an interactive computer. Hypertext is not a new idea. Bush [3] is credited with the first proposal for such a system, which he called the "memex," in 1945. Brown University's HES [5] and Engelbart's NLS [13, 14] are early implementations of computer-based hypertext systems, dating from the late 1960s. Only in recent years have hypertext systems become widely available. Commercially available systems such as Apple's HyperCard and Shneiderman's Hyperties [21, 25] may be purchased for use on personal computers. Research systems such as Xerox's NoteCards [17] show the wide range of components that

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are useful in a hypertext and the different presentations that help make it more understandable.\footnote{Two recent surveys of the hypertext field are [2] and [8]. In addition, [30] presents a perspective on the history and development of the area.}

Traditionally, a hypertext is composed of information fragments (such as text, graphics, sound, video) and tangible relationships among these fragments. One way to represent such a document is as a directed graph. Each information fragment is associated with a node in the graph, and the directed arcs represent the relationships among nodes. In this interpretation the arc represents a potential traversal from the source node of the arc to the destination node. A person browsing a hypertext traverses the graph and views (or hears) the information fragments as he visits nodes.

Some researchers are beginning to realize the need for a more formal substructure for the highly implementation-defined field of hypertext; for example, [12] and [16]. Projects that attempt to construct a more complete and descriptive mathematical basis for documents than directed graphs and annotations can provide are now beginning to appear.

The work described in this paper is based on one such mathematical framework—that provided by a Petri net. The Trellis model essentially provides a unifying formalism for describing and reasoning about many of the features of existing hypertext systems. The model also naturally expresses and manages concurrent browsing paths, an area of hypertext that has not been fully explored. Another advantage of this approach is that a Petri net is an incremental change to the commonly understood directed graph formalism, not a wholesale replacement of that representation, and results developed for directed graph-based hypertext are easily adapted to a Petri net formalism. Unlike a directed graph alone, though, a Petri net also permits the specification of a hypertext's \textit{browsing semantics}, that is, the dynamic properties of a reader's experience when browsing a document. This is due to the dual nature of Petri nets: they are easily and naturally represented as bipartite directed graphs, but they are automata as well, and have inherently parallel execution semantics. As automata they also have \textit{formal language properties} and can be viewed as \textit{language generators and recognizers}. These different but interchangeable aspects of nets provide analytical leverage for the solution of several interesting access control and version problems in hypertext.

Petri net formalism has long been recognized as an effective tool for describing and analyzing control flow among concurrent activities. Though we know of no other use of Petri nets in describing hypertexts, Petri nets have been used to model user interactions and interfaces. For example, Zisman [34] describes a model for activity coordination in office automation environments in which Petri nets are used to specify the possible interactions among a set of active agents. The agents themselves are represented as production systems that specify computational behavior. In another project, van Biljon [29] has used Petri nets to specify user interfaces, calling them man–machine dialogues. For illustration, he models a simple operating system shell. These practical uses of Petri net theory are similar in goals and approach to the Trellis model described herein; but our
work is cast directly into the hypertext domain and addresses some of the unique problems found there.

In the following section we present a brief review of Petri nets and then present our formal hypertext model. Following the presentation of the model, Section 3 describes aTrellis, a prototype Petri-net-based hypertext system. Section 4 then presents the solution to some specific hypertext problems as an illustration of the analytic power of our formalism. A discussion of useful extensions and further implications of the Trellis model, in Section 5, completes the report.

2. THE TRELLIS MODEL OF HYPERTEXT

The formal model of hypertext that we propose is based primarily on a Petri net representation of a document's structure. We take advantage of the fact that Petri nets not only capture the descriptive power of directed graphs, known to be a useful abstraction in hypertext systems, but provide as well a mathematically precise abstract machine for control and analysis of hypertext "execution" or "browsing."

For completeness of our discussion and to motivate the descriptive power of our approach, we first provide a short set of definitions for the Petri nets we employ. Following this, we give the definitions for our hypertext model. Readers familiar with basic Petri net theory can skip directly to Section 2.2 without loss of understanding.

2.1 Basic Petri Net Theory

To provide a complete presentation of the Trellis hypertext model, we review first the basic points from Petri net theory that are used in the later definitions. Readers who desire a more thorough exposition of net theory can consult the texts by Peterson [23] and Reisig [24]. The notation used here is taken from Reisig.

Definition 1. A Petri net structure is a triple, \( N = (S, T, F) \) in which

- \( S = \{s_1, \ldots, s_n\} \) is a finite set of places with \( n \geq 0 \);
- \( T = \{t_1, \ldots, t_m\} \) is a finite set of transitions with \( m \geq 0 \) and \( S \cap T = \emptyset \);
- \( F \subseteq (S \times T) \cup (T \times S) \) is the flow relation, a mapping representing arcs between places and transitions.

The arcs represented by \( F \) prescribe pre- and post-relations for places and transitions. The set of places that are incident on a transition \( t \) is termed the preset of \( t \), and is denoted by

\[ \cdot t = \{s \mid (s, t) \in F\}. \]

The set of places that follow a transition \( t \) is termed the postset of \( t \), and is denoted by

\[ t^* = \{s \mid (t, s) \in F\}. \]

The preset and postset for a place \( s \) are defined similarly as the sets of transitions incident on \( s \) and following \( s \), respectively.
In our model, we have simplified the notation often used for Petri nets by assuming that the weight on each arc is 1, and that the token capacity of each place is unbounded.

Definition 2. A marking $M$ of a Petri net structure $N = (S, T, F)$ is a function $M: S \rightarrow \{0, 1, \ldots\} \cup \{\omega\}$, mapping each place in the net into either a nonnegative integer or the symbol $\omega$. A marking is normally written as a vector $(m_1, m_2, \ldots, m_n)$ in which $m_i = M(s_i)$.

Each integer in a marking indicates the number of tokens residing in the corresponding place. The symbol $\omega$ represents an arbitrarily large number of tokens and is included for consistency with existing net theory. It may appear in Petri nets representing an infinite number of states.

An execution of a Petri net consists of a sequence of markings, beginning with the initial marking $M_0$ and ending in some final marking $M_f$ (which can be defined in several different ways, see [23]). To go from the current state $M$ to some next state $M'$, any transition $t$ that is enabled under $M$ is chosen and fired. For a transition $t$ to be enabled under $M$, it must be the case that $\forall s \in t: M(s) \geq 1$.

Firing $t$ consists of removing one token from each place in $t$ and adding one token to each place in $t^\ast$. The new state $M'$ is then the tuple of integers showing the number of tokens in each place after the firing.

For example, consider the marked Petri net shown in Figure 1. Places are drawn as circles, transitions as bars, and tokens as dots in places. The initial marking, shown in part (a), is $M_0 = (111000)$. The enabled transitions under $M_0$ are $t_1$ and $t_2$. If $t_1$ were chosen to fire, the resulting next state would be $M_1 = (001111)$, as shown in part (b). If, instead, transition $t_2$ were fired from the initial state, the resulting state $(100000)$ would be terminal from initial state since no further transitions would be enabled. Note that the number of tokens in $s_4$ is increased by 1 every time that $t_1$ is fired; hence $s_4$ acts as a counter.

Being a finite automaton, a Petri net has a dual mathematical nature. It can be viewed as generating (or representing) a formal language and as an abstract machine. From the formal language viewpoint, a Petri net describes a set of strings of symbols in which each symbol represents a transition in the net. From the automaton viewpoint, a Petri net is a state transition system in which the number of tokens in each place collectively constitutes the state of the abstract machine. Both views are of value to us when interpreting a Petri net in terms of a hypertext.

2.2 Formal Definitions of Trellis

The Trellis model of hypertext uses Petri net structure and execution semantics to specify both the linked form and the browsing semantics of a hypertext. This logical structure is then interpreted through a layer of indirection to arrive at a displayed form for reader consumption. Hypertext content and linked structure are effectively separated by the Trellis model.

Definition 3. A hypertext $H$ is a sextuple $H = (N, C, W, B, P_l, P_d)$ in which

- $N = (S, T, F)$ is a Petri net structure;
- $C$ is a set of document contents;
- $W$ is a set of windows;
- $B$ is a set of buttons;
- $P_l$ is a logical projection for the document;
- $P_d$ is a display projection for the document.

A hypertext consists of a Petri net representing the document’s linked structure, several sets of human-consumable components (contents, windows, and buttons), and two collections of mappings, termed projections, between the Petri net, the human-consumables, and the display mechanisms. A Petri net is a bipartite directed graph and, as such, is as capable of representing a linked hypertext as existing models using directed graphs alone. A window from set $W$ is a logically distinct locus of information. A button from set $B$ is an action that causes the current display to change in a specified way. A content element from the set $C$ can be many things: text, graphics, tables, bit maps, executable code, audio information, or, most important, another hypertext. Figure 2 shows an example of a Petri net structure for a hypertext. It will be used as the basis for further examples.

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2 Any directed graph has a simple representation as a Petri net; the bipartite nature of a Petri net is important for the execution semantics and does not restrict its structural modeling power. See Section 5.

3 The locus of information represented by a “window” is not necessarily visible. Indeed, a logical window may require other devices, such as an audio generator, for presentation of its information. Note that the model does not require that a logical window be associated with each net place (see the definition of logical projection, below). This is useful when a place is mapped to a computation that does not create presentable information (e.g., an action node in NoteCards).
Definition 4. Given the Petri net structure $N = \langle S, T, F \rangle$ in a hypertext $H$, the logical projection of $H$ is a triple $P_i = \langle C_i, W_i, B_i \rangle$ in which

\[ C_i: S \rightarrow C \cup \{\nu\} \]
\[ W_i: S \rightarrow W \cup \{\nu\} \]
\[ B_i: T \rightarrow B \cup \{\nu\} \]

A logical projection provides mappings from components of a Petri net to the human-consumable portions of a hypertext, as mentioned above. The $\nu$ in the definition is a special item denoting a null value. The function $C_i$ associates a content element with each place in the Petri net $N$. The function $W_i$ associates a logical window with each place in $N$ as well. The function $B_i$ associates a logical button with each transition in $N$. A transition is fired by selecting its associated logical button. Buttons can thus be thought of as the mechanism that couples hypertext browsing with Petri net execution. Together, these mappings provide an informational form and a logical layout for the browsing structure of the Petri net.

As mentioned above, content elements may be more than simple information. A Petri net place may have as its contents another hypertext, creating a natural hierarchy in the model. The uses for hierarchical structure in a hypertext are numerous. One obvious advantage is that each Petri net is independently analyzable. Many analysis algorithms for Petri nets are exponential in the size of the
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so a collection of smaller nets is preferable to one large net. Another advantage is that, with careful use of hierarchy, an author can organize a hypertext more effectively than with a flat structure. For instance, hierarchy can provide a form of abstraction. Browsing semantics can be designed so that the initial display of a lower-level hypertext shows an overview of its contents. The detailed text can then be browsed for more information, or it can be skipped by moving on at the upper level.

The display projection \( P_d \) is simply a collection of mappings that associate the logical buttons and windows of a hypertext with physical screen representations and locations. For example, given a text segment displayed in a physical window, one way to represent the logical buttons is to map them to a menu of mouse-selectable items located next to the text window; this is the approach in the current \( \alpha \)Trellis system, explained in Section 3 below. Another way is to map them onto words actually in the text window. Either mechanism (or others) can be used without altering the logical hypertext structure simply by appropriately specifying the display projection. Since our emphasis here is on logical representation and analysis, we do not present this aspect of the model in any detail.

There are, in summary, two important levels of indirection in this model. The first is the separation of content from structure. By appropriate selection of the logical mappings \( P_l \), one structure can serve to represent several document versions (see Section 4.5). The second level is the display projection \( P_d \), which allows a set of logical entities to be presented in different forms for consumption. For example, logical buttons that exist but are not selectable (due to the Petri net marking) can either be shown on a screen "grayed out" (as in the well-known Apple Macintosh menu format), or they can simply not be displayed at all. This second approach has implications for document security: If information is not attainable, its existence is not even admitted (see Section 4.4).

**Definition 5.** A marked hypertext is a pair \( H_M = (H, M) \) in which \( H = (N, C, W, B, P_l, P_d) \) is a hypertext, and \( M \) is a marking for the Petri net \( N \) in \( H \).

A marked hypertext can be thought of as representing the state of a hypertext during browsing. It is a characterization of the set of possible paths through a hypertext from a given point. A special case of marked hypertext, termed the initial state, is \( H_{M_0} = (H, M_0) \) where \( M_0 \) is an initial marking for the Petri net in \( H \). Different browsing patterns can be enforced on a single hypertext simply by choosing appropriate initial states, as demonstrated in the section below on access control restrictions. When a hypertext is first viewed, the node contents displayed correspond to the places that contain tokens in the initial net marking \( M_0 \); that is,

\[ \{ C_i(s) \mid s \in S \text{ and } M_0(s) > 0 \} \]

is the set of elements displayed.

The execution semantics of a Petri net provides the model of browsing a marked hypertext. A token in a place \( s \) indicates that the contents of the place \( C_i(s) \) are displayed for viewing (or editing, or some other interaction). When a token moves into an empty place, the associated content element is mapped to the display device; likewise, when all tokens are removed from a place, leaving it empty, its contents are removed from the display. Tokens move through the net.
as transitions are fired. This is accomplished by selecting logical buttons in the display. When a transition \( t \) is enabled in the Petri net, the logical button \( B_t(t) \) is mapped to some area of the screen where it may be selected.\(^4\) Browsing begins by starting execution of the Petri net in \( M_0 \). Browsing may terminate, depending on the structure of the hypertext, or it may cycle without end. If a state \( M \) of the Petri net is ever reached in which no transitions are enabled, then browsing ends, since no buttons are selectable at that point.

Hierarchy affects execution in the following way. When a token arrives at a place \( s \) having a marked hypertext as its value under the content mapping \( C_t \), the content elements corresponding to the places initially marked in the lower level net \( C_t(s) \) are displayed. Browsing in the lower level net continues concurrently with the remainder of the marked locations in the higher level net. The transitions leaving place \( s \) at the higher level remain visible for selection as long as they are enabled (this reflects the idea that, as long as the lower level net is being browsed, a token is sitting in the higher level place representing it). Selection of one of the buttons on arcs out of \( s \) causes immediate termination of the lower level browsing session. Otherwise, the lower level net may be browsed until no transitions in it are enabled. At this point, the reader has only the higher level transitions leaving \( s \) to select, and browsing continues at the higher level.

To summarize, a marked hypertext combines graphical structure with Petri net execution semantics to encapsulate all possible paths that a reader may follow through a document. During browsing, the current marking \( M \) of the Petri net determines which hypertext elements are viewable. The transitions enabled under \( M \) determine which buttons are visible, and hence selectable, in which windows. Selection of a button by the reader fires one of the enabled transitions, thereby generating a new state \( M' \) from \( M \) and causing the display to change correspondingly. Browsing formally terminates when no buttons are selectable.

3. THE \( \alpha \)TRELLIS HYPERTEXT SYSTEM

We have constructed a prototype hypertext browsing and authoring environment called \( \alpha \)Trellis to experiment with the Petri-net-based Trellis hypertext model. It presents a multipath browsing environment in which many different elements may be viewed at a given point in time. The current version of \( \alpha \)Trellis operates on Sun-3 workstations under the SunView window package. \( \alpha \)Trellis is intended as an experimental platform and a proof-of-principle vehicle. As such, we have paid more attention to implementing the Petri net document representation with browsing semantics, and have not implemented sophisticated window placement strategies.

\( \alpha \)Trellis allows both construction and viewing of a Petri net hypertext. Physically, the \( \alpha \)Trellis screen is divided into two main parts. On the right side is a Petri net editor and simulator, derived from Molloy's SPAN tool [22]. The left side of the screen is the hypertext browser, subdivided into four windows, each containing a text panel and a button panel. Using the net editor, an author builds

\(^4\) The choice of how to do this mapping is a design decision and is the function of the display mapping. The particular mapping chosen for the \( \alpha \)Trellis prototype is to show the button in the window of each place in the preset of \( t \). This is particularly useful in, say, a distributed implementation where the various windows may not appear on the same screen.

John Smith has worked for Megalith for 17 years and has been a productive worker the entire time. Early in his career upper management saw him as a strong candidate to move from technical work into policy making levels of management. His progress was slower than expected in the first five years, but his productivity picked up and now Mr. Smith has caught up to the early projected schedule and is believed ready to "make his mark" on Megalith Corporation.

Incidents requiring corporate discipline:
None.

Performance on corporate sports team was outstanding. Smith is generally a good team player, which can spell success for both him and Megalith off the field.
- Softball: 25 games played, 27 home runs, 47 RBIs.
- Bowling: average 187, season high 218
- Golf: handicap 15, season best 76
- Missed both company picnics.

Fig. 4. Browsing concurrent elements in cTrellis.
Fig. 5. Synchronization of parallel paths in αTrellis.
a Petri net structure and then, using tag strings, specifies the mappings of places to browsable text elements and transitions to buttons. The tag on a place is the name of a UNIX® file containing the associated text. The tag on a transition is the name displayed in a button panel when that transition is enabled to fire. During browsing, when a token resides in a place, the text in the file associated with that place is displayed in one of the four text panels. The place name appears in the upper left-hand corner of that window. Any enabled transitions in the postset of the place have their button names displayed in the button menu to the left of the text. Button names do not appear in any browser window until the associated transitions are enabled to fire.

Both the author and the reader can refer to the visual representation of the Petri net to get a graphical image of his location(s) in the document. Only an author is allowed to alter a document structure or directly place tokens in the net. Reading a hypertext is accomplished by simply executing the Petri net from its initial marking. Execution is controlled either (as is common in hypertext systems) by mouse selection of displayed buttons in the browser windows or by direct firing of enabled transitions from the net editor.

Figure 3 shows an initial screen from αTrellis, using the example shown earlier in Figure 2. This simple hypertext is a fictitious personnel record. Among other things, it contains salary, education, job history, and performance evaluation information for an employee. The Petri net structure of the hypertext is directly illustrated on the right side. The content mapping \( C_i \) is implicitly represented by the file names tagging the net places. The logical button mapping \( B_i \) is represented similarly by the tags on the transitions. The logical window mapping for this example simply associates a different logical window with each net place. The display projection implemented in αTrellis specifies that a logical window will be displayed in the first open panel on the left. If none are open, the extra logical windows are not displayed until some browsing action causes space to open up. The number of logical windows waiting for open space is noted at the bottom of the browser. In Figure 3, the button “evals” is being selected, causing the display shown in Figure 4 to appear. Notice that now there are three concurrently displayed elements. The net structure shows two different transitions leaving these places, and the buttons for these transitions are displayed next to each text panel. Here, button “recommend” in panel “review3” is being selected. Figure 5 shows the resulting screen display. All three previous windows have been removed together, and two more have been displayed as dictated by the net structure. Notice in these figures the token movement through the Petri net, corresponding to transitions fired by button selection.

4. SOLUTIONS USING THE MODEL

The Petri net basis for hypertext in the Trellis model allows the solution of several problems using state-space analysis techniques. In this section we discuss the following:

—Display parameters. Characteristics of the display, such as the maximum number of simultaneously needed windows, can be determined for planning screen layouts.

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* UNIX is a trademark of AT&T Bell Laboratories.

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—**Concurrent browsing paths and synchronization.** A Petri-net-based hypertext structure can naturally indicate that several content elements are to be viewed at the same time. More generally, an author can easily specify the creation and deletion of multiple concurrent browsing paths. These can be independent, or they can be related in various ways. An author can also specify that multiple paths be synchronized at certain places during browsing so that concurrently displayed information elements do not become unrelated to each other.

—**Node reachability and unreachability.** It is possible to verify that all nodes in a hypertext can be reached via some path; more important, verification is possible that certain nodes cannot be reached from particular initial markings, giving the basis for access control and for tailored versions.

Many of these solutions are derived from the reachability graph, a structure representing the various token distributions a Petri net may encounter during its execution. A marking can be thought of as the state of a Petri net automaton. The set of all markings a net can possibly attain during execution is called its reachability set. For a general Petri net, this set can be infinite, though for a Petri net representing a reasonable hypertext it is finite. In his book, Peterson summarizes a technique for representing this possibly infinite set as a tree that is guaranteed to be of finite size for any Petri net [23, p. 91, ff.]. The technique involves collapsing some infinite subsets of states into single metastates.

The left side of Figure 6 shows the reachability graph for the Petri net in Figure 2. Each node in the tree structure is a net state; the state number is indicated in the box and the marking for each state is listed in the table below the graph. Each arc leaving a node is labeled with the number of the transition that must fire to create the marking at the end of the arc. A node with a heavy line at its base is a duplicate of one found elsewhere in the graph. Note that the reachability graph is highly dependent on the initial marking. The right side of Figure 6, for example, shows the reachability graph for the same net structure as the one on the left, but with an initial marking \( M_0 = (0100000000000) \).

4.1 Display Complexity

Using the reachability graph for the Petri net in a hypertext, we can determine some parameters of the physical presentation that cannot be determined from a directed graph alone for many forms of hypertext browsing. One such parameter is the maximum number of windows that will be required for any reading of the hypertext. Since the model associates a content \( C_\ell(s) \) with each place \( s \) in the Petri net, if the browser displays each concurrently viewed element in a separate window, then the number of marked places in a Petri net state is the number of windows required to be concurrently displayed for that state. We can then scan the reachability graph node-by-node and find the maximum number of marked places over all states. For the reachability graph shown in Figure 6, this number is 5. This information can be employed in a number of ways: for example, it can aid the determination of a reasonable layout for a display mechanism that, say, tiles a screen with windows.

The analysis presumes that concurrently viewed elements are specified by the author rather than the reader. This means that in a single browsing session, a
reader is not allowed to simultaneously initiate separate traversals of the various alternative paths leaving a node. Systems allowing that form of browsing certainly exist. Rather than treating such behavior as a single session with arbitrary potential concurrency at each node, we view it as splitting one browsing session into several distinct sessions (or, equivalently, several logically distinct readers). The discussion of colored tokens in Section 5 provides a more detailed exposition of multiple readers in a hypertext.

It should also be noted that the Trellis model of hypertext neither prescribes nor prohibits any particular window placement or replacement strategy for concurrent displays. The particular choices of a hypertext system designer are reflected in the display mapping specified, but no form is inherently disallowed.
4.2 Concurrent Browsing Paths and Synchronization

As a computation model, Petri net theory is recognized as an excellent representation of parallel activities. As such, it is a natural choice for representing concurrent display of multiple elements and concurrent browsing paths in a hypertext. The advantage obtained over directed graphs alone is that the execution semantics of Petri nets provide an author with the opportunity to specify synchronization of concurrent activity.

There are several forms in which concurrent activity may appear. First is simply the display of several (presumably related) content elements simultaneously in different windows or via different media. For example, selection of a single button may cause a browser to see a text frame, a related picture, and perhaps hear some audio as well. These distinct elements are in essence a unit, and a single action should cause all three elements to be removed from the display when the reader is finished with them. This type of concurrent activity is created using a Petri net form in which a single transition (the button) branches to several places (the multiple elements) and a second transition is the sole output of these places. An example can be seen in the portion of Figure 2 composed of the transitions \( t_1 \) and \( t_2 \) along with places \( s_3, s_4, \) and \( s_5. \) The three content elements associated with these places will all be visible after button \( B_i(t_1) \) is selected. They will all be removed when either button \( B_i(t_2) \) or button \( B_i(t_{lo}) \) is selected.

As another example, consider the guided tours recently described by Trigg for Xerox’s NoteCards [28]. A guided tour is formed from a pool of possible display windows (“cards”). The various cards of interest to an author are collected into sets called “tabletops.” The cards in a tabletop are all displayed concurrently. An author constructs a tour by linking tabletops together to form a directed graph. From any one tabletop there may be several alternative next tabletops, and the final linked structure may be cyclic as well.

Guided tours are a subclass of the hypertexts one can describe with Petri nets. The browsing control they offer can be represented by the Trellis model in two ways. The first method does not use hierarchy; rather, we assume that each place in a Petri net has an information element associated with it. A tabletop is then represented by a Petri net fragment consisting of a single transition connected to as many places as there are elements in the tabletop (with the exception of the first tabletop, which has no transition). Tabletop net fragments are interconnected as desired by the author. When one tabletop is connected to another, an arc is created from each of the places in the source fragment to the single transition in the destination fragment. This construction is illustrated by the left side of Figure 7. Note that lone transitions may follow terminating tabletops (for instance, transition \( t_s \) follows places \( s_5 \) and \( s_6 ) \) to clear their elements from view at the end of a tour. To begin browsing, a token is placed in each of the places constituting the first tabletop of the tour.

The second construction technique uses the model’s hierarchy, and it is more illustrative of the underlying sequential finite automaton in guided tours. The set of elements in a tabletop is fixed, so we represent each tabletop as a simple lower level Petri net: one place for every information element in the tabletop, with no connected transitions. Then, at the higher level, a single place followed
Fig. 7. Petri net representation of guided tours: (a) without hierarchy, (b) using the hierarchy.

by a single transition will represent each tabletop (again with the exception of
the first tabletop). A tabletop is linked by an author to others as in the previous
method. The right side of Figure 7 shows this construction. The semantics for
browsing a net hierarchy specify that a token arriving at a high-level tabletop
place causes all the places in the low-level net to be marked, and their contents
thereby to be displayed. Since the lower level net representing a tabletop has no
transitions, no browsing can occur at the lower level. The only button selections
a reader can make correspond to the high-level transitions that leave the high-
level place representing the entire tabletop. For example, when browsing the
tabletop represented by place $s_3$, content elements for places $s_{3.1}$ and $s_{3.2}$ are
visible and buttons for transitions $t_4$ and $t_5$ selectable.

Since the high-level Petri net formed by the hierarchical construction method
has a single input and a single output arc for each transition (the terminating
transitions $t_3$, $t_6$, and $t_8$ can be thought of as all leading to a single "final state"
place with null contents), it is equivalent to a deterministic finite state machine
[23, p. 41, ff.]. Thus, separate parallel browsing paths in a guided tour must be
simulated unnaturally. Careful selection of the elements that appear in successive
tabletops can give the appearance of parallel independent paths, but such paths
cannot be identified from the linked structure alone.

Therefore, we introduce a second form of browsing concurrency—a generali-
zation of multiple concurrent elements to multiple concurrent paths. Concurrent
browsing paths are two or more sequences of content elements that are displayed
at the same time. During browsing, concurrent paths may be synchronized and
coalesced, they may join and overlap but still remain distinct, or they may never

join again after their creation. The degree of relatedness among elements in two different paths can vary from complete independence to a strong dependence requiring rigid and frequent synchronization.

Separate concurrent browsing paths exist whenever multiple concurrent elements are created, and they remain distinct until they are synchronized and coalesced. When several parallel paths are active, one content element from each is concurrently visible, but the speed of browsing along each path can vary according to the desires of the reader, within the limits prescribed by the structure. The author specifies the desired parallel path interactions based on an understanding of what net execution semantics allow and what the content and context of the information require. Thus, a browser’s experience with a Petri-net-based hypertext can be somewhat under the author’s control.

Different parallel browsing effects are easily created by varying the placement and frequency of synchronizing events in a Petri net structure. Figure 2 contains some simple paths that are independent for a while, but then are synchronized and coalesced. Firing transition \( t_4 \) causes two concurrent paths to be created, showing first elements \( C_i(s_i) \) and \( C_i(s_5) \). Instead of having both elements immediately removed thereafter, browsing continues along the rightmost path only. Firing transition \( t_5 \) causes the content element of place \( s_6 \) to replace the element for place \( s_7 \), while the element for place \( s_7 \) remains visible. Transition \( t_6 \) is not enabled for firing (and, therefore, the button \( B_i(t_6) \) is not visible for selecting) until after transition \( t_5 \) is fired. Firing \( t_6 \) then removes both content elements from view and coalesces the two browsing paths into one. This example is short, but the principle generalizes to arbitrarily long paths with numerous parallel branches. Even though browsing activities are concurrent on the different paths, they are not allowed to progress at speeds that are too dissimilar. In essence, synchronization points required the reader to “catch up” on each of the multiple paths before continuing on any of them.

A synchronized structure like this can be used for presenting information simultaneously at several levels of detail or in several complementary forms. For example, using only the net structure in Figure 2, the content mapping could be one that maps place \( s_7 \) to, say, a picture of the NASA space shuttle. The content elements associated with places \( s_6 \) and \( s_7 \) could then be textual descriptions of some aspect of the picture, say an explanation of the orbiter vehicle itself and a description of the booster rockets, respectively. The Petri net structure shown dictates that the text descriptions are to be displayed in sequence and the picture of the shuttle is to remain visible during the entire sequence. When the text frame sequence is consumed, selecting the single button \( B_i(t_6) \) would remove all the related concurrent windows from view. The burden of cleaning up multiple displays can thus be removed from the reader by the synchronization inherent in a Petri net automaton.

Concurrent browsing paths do not have to be synchronized or coalesced. They can overlap like a pipeline, join and separate, or just individually come to an end. Figure 8 shows a Petri net structure in which four parallel browsing paths are active. From an initial marking \( M_0 = (1000000 . . .) \), transition \( t_1 \) is fired to create

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1 This example was suggested to us by a novel graphic display of the NASA space telescope in an experimental Sun workstation version of Shneiderman’s Hyperties system [21, 25].
the next marking $M_1 = (0101110 \ldots)$. Transition $t_2$ is then fired to create the marking $M_2 = (0011110 \ldots)$ shown in the figure. The contents of the four places $s_3, s_4, s_5,$ and $s_6$ are all concurrently visible. From this point, the reader can select three different buttons. Selection of button $B_i(t_3)$ will cause the leftmost path to simply terminate, removing the contents of $s_5$ and replacing it with nothing. Selection of button $B_i(t_5)$ will instead synchronize the middle two paths and move them further along in the net. Selection of button $B_i(t_4)$, though, is perhaps the most interesting of these cases. This action will create a net marking of $(0010121 \ldots)$, putting a second token in place $s_6$. The rightmost path continues to exist separately from the middle two; they are not coalesced into a single path, as can happen at a synchronization point. The paths simply come together and overlap here. Even with two tokens present in $s_6$, its content element is displayed in only one spot. When either of transitions $t_5$ or $t_6$ is fired, though, a token will remain in $s_6$ so its contents will remain displayed. One path will move on and another will remain at $s_6$.

Note that, in this example, buttons can come and go within the context of one content element. When $C_i(s_6)$ is displayed, for instance, there may sometimes be only one button shown (for $t_5$), as in the scenario of Figure 8. At other times two buttons may be shown for this same element. For instance, from marking $M_1 = (0101110 \ldots)$ transition $t_4$ can be fired to create a next marking $M_2 = (0100111 \ldots)$. In this state, both transitions $t_5$ and $t_6$ are enabled so their corresponding buttons will both be displayed with the contents of place $s_6$. In this particular marking, the button for $t_6$ will also appear in another window with the content element for place $s_6$.

An example of a hypertext that might effectively use multiple browsing paths is one in which the reader needs to accomplish several independent tasks, say

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Footnote 6: This nonstatic link behavior is an important feature of van Dam and Feiner’s Document Presentation System [15], which is further described by van Dam [30, p. 894].

locating a record in a personnel file and locating a record in a payroll account. The relative speeds with which the browsing tasks are accomplished are unimportant since they are essentially independent.

A variant of the multiple-path form of concurrency is obtained using a Petri net extension called colored tokens [19]. Here, each token is associated with one of several classes, and firing a transition requires all input places to be marked with like-colored tokens. By using this model, completely independent browsing sessions can be maintained. For example, a single hypertext can be shared and browsed by different readers without them interfering with each other. Each reader is given a different token color. More discussion on the use of colored Petri nets in hypertext can be found in Section 5.

4.3 Node Reachability and Unreachability

As with directed graphs alone, the Petri net model can be used to determine if portions of a hypertext can actually be reached during browsing, or alternately, if portions can never be reached. The latter consideration forms the basis for a unified method of providing hypertext access control, as discussed in the next section. Given a hypertext $H$, an initial marking $M_0$, and a place $s$, to determine if a particular content element $C_i(s)$ can be viewed during browsing, one must simply compute the reachability graph for the marked hypertext $(H, M_0)$ and then scan the nodes, looking for a state in which place $s$ is marked. If no state exhibits such a marking, then the information cannot be viewed when the hypertext is browsed starting in state $M_0$. Similarly, we can determine if certain collections of information can be viewed simultaneously by looking for states containing sets of marked places.

Another characteristic of a hypertext that can be determined from the reachability graph is termination. If a state $M$ exists in which no transitions are enabled, then there can be no next state $M'$ from $M$, so the browsing session in that Petri net must terminate in $M$. Such terminal states appear in the tree representation of a reachability graph as leaves that are not duplicates of other nodes. If the author desires that a hypertext have no terminal states, then this property can be easily checked by scanning all nodes in the graph.

Similarly, an author may wish to verify that any terminal states in a browsing session are ones in which the screen is blank, that is, all places are unmarked in a terminal state. Another state property that is reasonable to look for is that a browsing session can return to its initial state, thus making it cyclic. Any characterization one wishes to describe for a marking can be checked against the reachability graph in order to verify that a hypertext has, or fails to have, a particular property.

Another class of properties that can be checked for a browsing session are temporal relationships among visitations of content elements. A Petri net, being an automaton, can be viewed as a language generator. The language of a Petri net [23, p. 151, ff.] is the set of all sequences of transitions that can be fired during execution. In hypertext terms, the corresponding concept is the button sequences a reader might select during browsing from an initial marking. Consequently, analysis of the language of the Petri net can be used to verify the browsing requirements an author may have for a hypertext. For example, an author may wish to ensure that any browsing session that encounters an element.
Y must have somewhere previously encountered element X (e.g., X is a legally required statement that protects trade secrets divulged in Y). Another example is ensuring that any browsing session encountering an element X cannot subsequently encounter element Y, but a browsing session that first encounters Y can later encounter X (e.g., X is an expert-level explanation of a topic and Y is a novice-level explanation of the same topic). These are difficult properties to verify by inspection in a complicated net.

4.4 Access Control

The Petri net formalism can be effectively used to enforce browsing restrictions on readers of a hypertext. In a hypertext represented as a plain (unannotated) directed graph, if two browsing paths share a common node, then no reader can be prevented from visiting other nodes that lie on paths leaving that common node. With an unannotated Petri net, however, browsing paths can share numerous common subsections and still have interspersed mutually exclusive sections as well. The limitation of plain directed graphs is usually dealt with in hypertext systems by annotating arcs with keywords or attributes and controlling arc access via predicates. The Neptune system [11], for instance, uses a list of attribute/value pairs on nodes and arcs to provide conditional browsing of hypertext sections. Attributes in Neptune can be dynamically created and deleted by readers, a feature that the Trellis model does not include because of a stronger distinction between a hypertext's author and its readers.

The access control capabilities described in this section are otherwise not different from those available with graph augmentation techniques such as tagged arcs. However, we believe the Petri net notation is as intuitively easy to use as existing augmentation techniques, if not easier, while being more succinct and convenient. The notation successfully describes different existing access control mechanisms in one unified mathematical formalism, making them accessible to comparative reasoning and analysis.

With techniques like tagged arcs, the manner in which the graph additions are used by a browser is not inherently obvious. The exact semantics of their behavior and capabilities depend on the hypertext implementation and how it employs the added notation. With the Petri net model, access capabilities become an integral part of a document, as opposed to being properties of the browser. What this really implies is that a Petri net engine essentially provides a standardized abstract browser implementation, one whose semantics are mathematically defined, as opposed to being "code defined." The unified execution abstraction of access control integrated with document structure is an important difference from augmentation techniques like tagged arcs and predicates.

Access control in the Trellis model uses the idea of a marked hypertext. For a hypertext \( H \), various classes \( U_i \) of users can be identified, depending on which portions of \( H \) the author desires to be visible to certain readers. Then each class can be given a different starting state \( M^i_0 \) for browsing \( H \). Thus, each class \( U_i \) constitutes a separate marked hypertext \( H_{M^i_0} = (H, M^i_0) \).

7 The Hypertext Abstract Machine [4], the basis of the Neptune system, is a similar attempt to provide an abstract system semantics for standardization of browsing behavior.

For example, consider a scenario in which an employment record will have two classes of reader: privileged (class P) and restricted (class R). Readers of class P can browse the entire document, but readers of class R may not see the sections containing, for example, job performance evaluations. We refer again to the Petri net structure in Figure 2 to represent this scenario. Let places $s_3$ through $s_5$ have job evaluations as their images under mapping $C_i$; as discussed earlier, the structure specifies that the three are to be displayed concurrently. Places $s_{12}$ and $s_{13}$ also contain privileged information, and their contents are to be displayed simultaneously after the evaluations have been read. The rest of the employment record ($s_2$, $s_6$ through $s_9$, and $s_{11}$) is unrestricted information. Places $s_1$ and $s_{10}$ are special “access control” places that serve to separate the classes of reader during browsing. On this document, readers of class P define a hypertext with initial Petri net marking $M_P^0 = (110000001000)$, whereas readers of class R define a hypertext with initial marking $M_R^0 = (010000000000)$. The initial marking shown in Figure 2 is that of class P. The reachability graph resulting from this marking is shown in Figure 6(a). Having a token in each of places $s_1$ and $s_2$ enables transition $t_1$. This in turn allows readers of class P to select the button $Bl(t_1)$ and thereby read the job performance evaluations. Readers of class R do not start with a token in the access control place $s_1$; a scan of the reachability graph for their initial hypertext state, shown in Figure 6(b), shows that no token can ever be there. Since transition $t_1$ is never enabled for readers of class R, the evaluations can never be browsed.

Readers of class P can also choose to leave the restricted portion of the hypertext by selecting the button $Bl(t_{10})$ after reading the evaluations at places $s_3$ through $s_5$. The token in access control place $s_{10}$ allows them to reenter the restricted section at a later point. Firing transition $t_7$ replaces this access control token. Class R readers will never have a token in place $s_{10}$, so they will always be excluded from sections $s_{12}$ and $s_{13}$ of the hypertext (again, see Figure 6(b)). Note that for both access control places in this example, the net structure is such that firing the transitions they guard replaces their respective tokens.

A third class of reader can be defined on this hypertext: semirestricted (class S). Readers of class S can see the salary information contained in places $s_{12}$ and $s_{13}$, but they cannot see the performance evaluations in places $s_3$ through $s_5$. This restriction can be created by giving readers of class S an initial marking $M_S^0 = (0100000001000)$. An examination of the reachability graph for this marked hypertext shows that the token in access control place $s_{10}$ will allow a reader to fire transition $t_7$, but that transition $t_1$ can never be enabled.

The structure of this simple example suggests the following strategy for constructing hypertexts with access control classes. First, outline the access control classes for the hypertext. Next, construct a collection of subhypertexts, each having a separate Petri net structure and each intended to be read in its entirety by access control class members. Next, for each subhypertext, decide which of the classes can browse it. Finally, using the hierarchy in the Trellis model, construct a top-level hypertext that has one place representing each subhypertext and that is connected with the desired browsing patterns. Add one access control place for each subhypertext place, connected like the ones in Figure 2, as input and output to the transitions before and after the subhypertext place.

For each access control class, prescribe an initial marking that includes a token in the access control place of every subhypertext the class is allowed to browse.

4.5 Tailored Versions of a Hypertext

Tailored versions of a hypertext are useful when each is to contain information specific to a separate environment. Portions of the contents of the documents in a collection of such tailored versions are common to all of the versions, and other portions are specific to one or to a subset of the documents. An example of a situation in which a tailored document set would be useful is to describe the invocation and use of a piece of software that runs on a number of different computers or operating systems. A similar situation is to support different user groups with different kinds of expertise (for example, a spreadsheet might be described for a user community that was expert in computer use or alternately for a group that was expert in accounting principles).

While each version could be maintained as a separate hypertext, a more attractive design is to take advantage of the commonality of structure and content in the collection of tailored versions and to represent the collection as a single Petri net. The issue of representation and of specification of appropriate alternate choices for content is exactly the same issue discussed in the previous section on access control in browsing, and the same solutions apply. The reachability graph analyses needed to verify the correctness of tailored versions may differ, however, from those defined for browsing control. One property to be shown for tailored versions is that exactly one of a set of places is reachable (i.e., the content element for only one of the versions is ever viewable). A second property to be shown is that every place in a particular set is reachable (i.e., all the contents of one version are potentially viewable).

It is interesting to note that tailored versions of a hypertext may also be specified through modification of the function $C_i$. The mapping defined by $C_i$ would be the same for all common portions of the two versions, but would differ for the changed content elements. For example, an author may wish to create two parallel versions of a document, one written in English and the other in French. In this case, two different mappings would be associated with the Petri net places: one to the English-language contents, the other to the French-language contents. In this case, there would be no overlap between the content elements in the two versions. Similar substitutions may be used to tailor versions of a document for different environments—for example, a user manual describing software that runs on a number of different pieces of hardware, or perhaps a descriptive document with versions at different levels of technical complexity corresponding to the technical knowledge of differing communities of readers. In these cases, only a subset of the content elements would vary among different versions.

Similar effects have been studied in the context of traditional documents using attribute-value pairs as the means for identifying the differing alternatives (for example, see Ilson's recent paper [18]). Within the context of the Trellis model, it is possible to verify that two versions based on the same logical structure contain corresponding collections of elements and exhibit the same connectivity. Browsers based on the Trellis model can easily be designed to
permit simultaneous viewing of the separate versions and to provide additional tools to help an author ensure that two versions are consistent with each other.

5. DISCUSSION AND IMPLICATIONS OF THE MODEL

Consider again the common representation of a hypertext as a (usually annotated) directed graph in which a single element is visible at any time during browsing. The Trellis model does not restrict this representation; rather, it generalizes it with concurrency and synchronization. This claim is supported by the observation that any directed graph can be expressed as a Petri net by a well-known simple transformation [23, p. 41, ff.]. The transformation takes the directed graph that represents such a hypertext, makes each existing node a Petri net place, and creates a Petri net transition in the middle of each arc. Nets constructed by this technique are in an equivalence class with finite state machines. The browsing semantics under our model for the hypertexts are exactly as expected for the original directed graph form. Thus, methods applicable to directed graphs, such as arc annotations and history mechanisms, are equally applicable to Petri-net-based hypertexts.

In addition to directly representing graph-based documents in Petri net form, a system based on the Trellis model can also adopt many of the user interface techniques that have been developed for browsing directed graph hypertexts. The Petri net provides the same logical document structure that a directed graph does in a browsing tool. The results of empirical studies into visual aspects of a hypertext browsing system (for example, see Shneiderman's paper [25]) are relevant when applied to either representation. However, research is needed into effective window placement strategies for multiple concurrent elements.

Several points about Petri net representation deserve further emphasis. First, the Trellis model provides the author of a hypertext with greater control over the sequences in which nodes are browsed. Other researchers have found a similar need to permit this kind of specification (both within one document and among multiple documents [6, 7, 32, 33]). However, the Petri net model permits flexible enforcement of such sequencing, making browsing restrictions an integral part of a hypertext's structure rather than applying it with an external browsing mechanism. As such, we believe the Trellis model provides a needed unified structuring facility to the author. Furthermore, as with other browsing control facilities, the control provided by Petri nets is not forced on an author. The fact that unrestricted directed graphs (i.e., finite state machines) are a subclass of Petri nets implies that an author can choose to apply no browsing control, if that is the effect desired. Concurrency and sequencing control are available, though, and can be specified to whatever degree an author requires. Finally, augmentation techniques that are applicable to directed graph hypertexts are equally applicable to Petri nets. Essentially, a hypertext system based on highly annotated directed graphs can have its document representation replaced by Petri nets, thereby gaining concurrency specification and control without sacrificing any existing functionality.

Two other advantages of the Trellis hypertext model should be noted. First, the Petri net representation is essentially graphical and, consequently, is only an
incremental change from the common directed graph model of hypertext. Second, Petri nets have been studied and analyzed for over twenty years, so an extensive theory exists that can be immediately applied to the solution of problems in the hypertext domain. Indeed, useful extensions have been defined to Petri nets that will have direct application to hypertext as well. Two such extensions are deterministically timed Petri nets and colored Petri nets.

Use of a deterministically timed Petri net in the model allows hypertext documents such as timed exams and paced presentations, or even simple animations. Several types of timed Petri nets have been well developed, and a body of results is available for determining such temporal properties as relative firing frequencies of transitions or execution durations of subnets. In the timed Petri net model that seems most appropriate for hypertext, an integer is associated with each place in the Petri net; it represents the number of time units (clock "ticks") that must pass before a token in the place can enable any of its output transitions to fire. In hypertext terms, the integer time on a place indicates the minimum amount of time that the contents of a place are displayed when a token arrives at the place. After the requisite number of ticks has passed, the token would participate in enabling transitions for firing, meaning either that selectable buttons would appear in display windows, or, if timing were to be strictly enforced, transitions would be fired automatically, thereby causing the displays to change.

Colored Petri nets extend the classical model by associating colors with the tokens, the places, or perhaps the transitions of a net. A color is simply a method of distinguishing classes of elements that share the same structural category. The firing rule is altered to require some color property to hold, in addition to having tokens in all input places for an enabled transition. For example, the rule may require all tokens being consumed by a transition to be of the same color, or to be all of the same color and to be the color of the transition. Color extensions increase the size of the state space that can be represented with a given net structure.

Several applications of colored Petri nets are of note in the hypertext domain. In one obvious extension, colored tokens permit multiple, simultaneously active, asynchronous browsing sessions over a hypertext. Additional sessions cannot be created by simply adding tokens to an uncolored net because this could permit traversal of otherwise prohibited paths. In a colored net, a new independent browsing session would be generated by replicating the initial marking for the net (or perhaps the current marking) with a previously unused token color. This preserves the correctness of the author-specified net behavior because tokens of

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8 Contrast the use of this graphically based model with a nongraphical one such as Garg's, which is based in abstract algebra [16]. While the nongraphical models have certainly demonstrated their utility, the mental shift required to employ their results is significant.

9 See, for example, the proceedings of the 1985 International Workshop on Timed Petri Nets and the proceedings of the 1987 International Workshop on Petri Nets and Performance Models.

10 Dami et al. [10] describe a general timed computation mechanism called temporal scripts that has similar goals, in that animated graphical presentations are principal examples in their paper. The approach in temporal scripts is to allow an object to consume virtual time as a resource and to have time slices passed around from agent to agent to effect computation. In lieu of timing on a Petri net, this model appears to be one that can also be used in conjunction with Trellis to specify and enforce more precise temporal properties for events during hypertext browsing.

differing colors do not interact. The issues surrounding the actual mapping of separate browsing sessions to a display, or a network of displays, are outside of the scope of this paper, and indeed schemes such as those of Lifshitz and Shneiderman [20, 21] could be adopted to provide this partitioning.

Another application for colored Petri nets is to permit definition of additional levels of granularity in access control. KMS [1, p. 832] has considered the case of hypertext nodes that are both readable and modifiable by browsers (for instance, to permit addition of notations and commentary). At issue is how to grant some browsers the right to modify while restricting others to reading only. One approach might be to encode the access rights in the token's color and to generalize the logical projection \( P_l \) and the display projection \( P_d \) to associate an appropriate presentation with each color. Browsers receiving the appropriate color of marked hypertext would then be granted modification privileges as they browse.\(^{11}\)

6. CONCLUSIONS

In summary, we have presented a Petri net model of hypertext that generalizes the directed graph models currently in use. Since a Petri net is a directed graph, the Trellis model specifies the logical linked structure of a hypertext; being automata as well, Petri nets provide standardized browsing semantics for hypertext systems. As a natural formalism for concurrency, Petri net notation also provides a unique mechanism for the expression and control of concurrent browsing paths in a hypertext. This browsing control can be applied to or excluded from a document at the author's discretion. Since the finite state machine representation used in many hypertext systems is given by a subclass of Petri nets, useful annotations like tagged arcs and predicates can be as easily applied to the Trellis model as they can to directed graphs. The Trellis model, then, provides a unifying formalism for describing and reasoning about browsing behavior, both concurrent and sequential, both controlled and unrestricted.

An important research problem now is the development of composition techniques to produce "well-formed" documents and the incorporation of these techniques into a comprehensive authoring and analysis environment. One example of such an approach is the heuristic mentioned earlier for constructing a hypertext with access control classes. Another approach is an authoring language that we envision to structure meaningful hypertexts by the repeated composition of a few simple Petri net fragments, much as control structures in modern programming languages alleviate the confusion of an unstructured assembly code program [26].\(^{12}\) The authoring language not only ensures that properly structured documents are produced, but the procedural programming paradigm hierarchically organizes a hypertext and helps manage the complexity of constructing and analyzing a very large document. An authoring environment

\(^{11}\) We note that this use of colored tokens is a special case of capability-based addressing as defined in the HYDRA operating system [31]. The more general statement is that we associate a capability with the token and that the actions carried out in the \( P_l \) and \( P_d \) mappings can be defined based on that capability.

\(^{12}\) Van Dam makes a similar point in his call for identification of new flow-of-control kinds of hypertext constructs [30, p. 894].
based on the Trellis model and this structured programming approach to document construction must provide tools for writing in the language, as well as analysis tools for verifying that a hypertext will behave as expected.

The merging of hypertext and Petri nets is proving to be an advantageous combination. Much of the power of the model, and corresponding implementation, results from our ability to adopt already established user interface techniques from the hypertext community and analytical techniques from the Petri net community. We believe that this format model will allow the development of hypertexts that are not only general but also predictable in their behavior.

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